Math 115

Summer 2017

Lecture 1

Ch. 1 order of operations, Properties of real numbers, evaluating expressions, and Some basic translation

Order of operations:

1) Do inside of groups: ( ), [ ], { }, | |, —, √‾, ...

2) Do exponents & roots

3) Multiplication and division from L2R.

4) Addition & Subtraction from L2R.

Simplify:

1) $36 \div 3^2 - 2^2 = 36 \div 9 - 4$

$= 4 - 4$

$= \boxed{0}$ Do not use $\emptyset$ for zero.

2) $\sqrt{10^2 - 8^2} - 3 \cdot 2 = \sqrt{100 - 64} - 3 \cdot 2$

$= \sqrt{36} - 3 \cdot 2$

$= 6 - 3 \cdot 2$

$= 6 - 6 = \boxed{0}$

3) $\dfrac{|-5 \cdot 2|}{-\sqrt{4^2 + (-3)^2}} = \dfrac{|-10|}{-\sqrt{16+9}} = \dfrac{10}{-\sqrt{25}} = \dfrac{10}{-5} = \boxed{-2}$

4) $(10^2 - 5 \cdot 2^2) \div (\sqrt{169} - \sqrt{144})$

$= (100 - 5 \cdot 4) \div (13 - 12)$

$= (100 - 20) \div (1) = 80 \div 1 = \boxed{80}$

$\sqrt{\ }$ Square root. $\boxed{\sqrt{\ }} = \text{Ans}$ $\text{Ans}^2 =$

5) $\dfrac{7(5\cdot 2-3^2)}{2^4-(-4)^2}=\dfrac{7(10-9)}{16-16}=\dfrac{7\cdot 1}{0}=\dfrac{7}{0}$

$(-4)(-4)=16$

$\dfrac{\text{Nonzero}}{\text{Zero}}=\phi$

$=\text{undefined}$

$=\boxed{\phi}$

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6) $\left(\sqrt{\dfrac{16}{25}}-\sqrt{\dfrac{49}{100}}\right)\div\left(-2\dfrac{2}{5}\right)$

$=\left(\dfrac{4\cdot 2}{5\cdot 2}-\dfrac{7}{10}\right)\div\left(-\dfrac{12}{5}\right)=\left(\dfrac{8}{10}-\dfrac{7}{10}\right)\div\left(\dfrac{-12}{5}\right)$

LCD=10

$=\dfrac{1}{10}\div\dfrac{-12}{5}=\dfrac{1}{10}\cdot\dfrac{-5}{12}=\boxed{\dfrac{-1}{24}}$

2

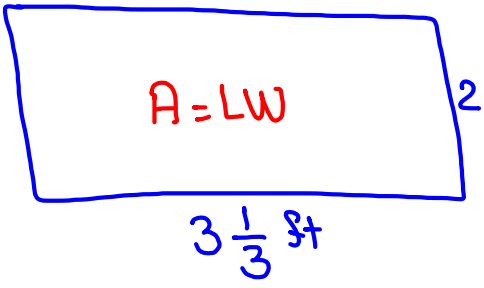
8) $\dfrac{10}{45}+\sqrt{\dfrac{25}{49}}=\dfrac{5\cdot 2}{5\cdot 9}+\dfrac{5}{7}$

$=\dfrac{2\cdot 7}{9\cdot 7}+\dfrac{5\cdot 9}{7\cdot 9}$

LCD=9·7=63

$=\dfrac{14}{63}+\dfrac{45}{63}=\boxed{\dfrac{59}{63}}$

Find the area



$A = LW$

$= 3\frac{1}{3} \cdot 2\frac{1}{4}$

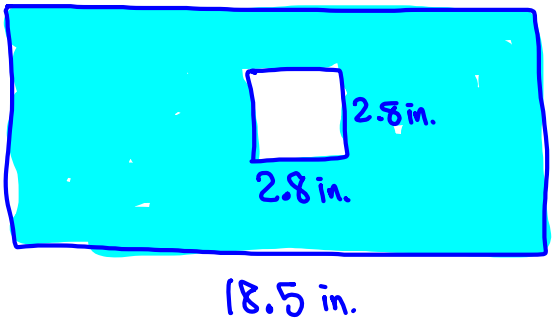
$= \frac{\cancel{10}^{5}}{\cancel{3}} \cdot \frac{\cancel{9}^{3}}{\cancel{4}_{2}}$

$= \frac{15}{2}$

$A = 7\frac{1}{2}\ ft^2$

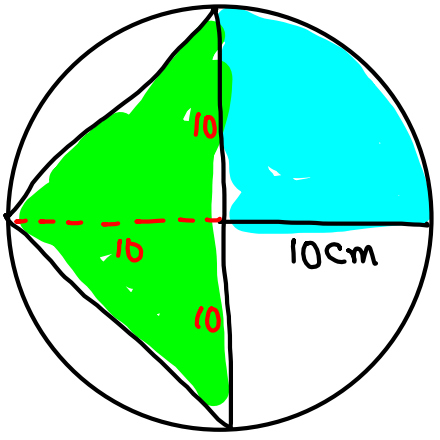
Find the Shaded area:

$A = A_{Rectangle} - A_{Square}$



$= (18.5)(7.2) - 2.8^2$

$= 125.36\ in^2$



Find shaded area

$$A = \frac{A_{circle}}{4} + A_{Triangle}$$

$$A = \frac{\pi r^2}{4} + \frac{bh}{2}$$

$$= \frac{(3.14)(10)^2}{4} + \frac{20 \cdot 10}{2}$$

$$A = 178.5 \text{ cm}^2$$

Simplify:

$$\frac{3}{5} - \frac{1}{2} - \frac{7}{10} = \frac{3 \cdot 2}{5 \cdot 2} - \frac{1 \cdot 5}{2 \cdot 5} - \frac{7}{10}$$

LCD = 10

$$= \frac{6}{10} - \frac{5}{10} - \frac{7}{10} = \frac{-6}{10}$$

$$= \boxed{\frac{-3}{5}}$$

Simplify

$$\left(\frac{8}{15}-\frac{7}{10}\right)\cdot\left(1\frac{1}{4}-3\frac{1}{3}\right)$$

LCD=30

$$=\left(\frac{8\cdot2}{15\cdot2}-\frac{7\cdot3}{10\cdot3}\right)\cdot\left(\frac{5\cdot3}{4\cdot3}-\frac{10\cdot4}{3\cdot4}\right)$$

LCD=2

$$=\left(\frac{16}{30}-\frac{21}{30}\right)\cdot\left(\frac{15}{12}-\frac{40}{12}\right)=\frac{-5}{30}\cdot\frac{-25}{12}=\boxed{\frac{25}{72}}$$

Simplify:

$$\frac{\cancel{10}^{2}}{\cancel{21}_{3}}\cdot\frac{\cancel{14}^{2}}{\cancel{15}_{3}}-\frac{2}{3}$$

$$=\frac{4}{9}-\frac{2\cdot3}{3\cdot3}=\frac{4}{9}-\frac{6}{9}=\boxed{\frac{-2}{9}}$$

LCD=9

Mathematical Expressions

It is a combination of numbers, operations, and letters (Variables).

$5x+3$, $2x^2-7x-4$, $\sqrt{x^2+y^2}$, $\frac{x^2-4}{x^3-8}$

For now, we do a couple of things with expressions

- Evaluate
- Simplify

Evaluate

$-2x^2-5$ for $x=-4$.

$=-2(-4)^2-5$

$=-2(16)-5 \quad =-32-5 \quad =\boxed{-37}$

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x^3-y^2 for $x=-4$, and $y=8$.

$=(-4)^3-(8)^2 \quad =-64-64$

$=-64+(-64)=\boxed{-128}$

$$\frac{x^2-4}{x^3+8} \quad \text{for } x=-2.$$

$$= \frac{(-2)^2-4}{(-2)^3+8} = \frac{4-4}{-8+8} = \frac{0}{0} \quad \text{indeterminate}$$

$\frac{\text{Zero}}{\text{NonZero}} = \text{zero}$, $\frac{\text{NonZero}}{\text{Zero}} = \text{undefined}$

$\frac{\text{Zero}}{\text{Zero}} = \text{indeterminate}$

$$\frac{\sqrt{x^2+y^2}}{x-y} \quad \text{for } x=-3, \& y=-4.$$

$$= \frac{\sqrt{(-3)^2+(-4)^2}}{(-3)-(-4)} = \frac{\sqrt{9+16}}{-3+4} = \frac{\sqrt{25}}{1} = \frac{5}{1} = \boxed{5}$$

$$\frac{2x-3y}{\sqrt{y^2-4x^2}} \quad \text{for } x=-5,\ y=10.$$

$$= \frac{2(-5)-3(10)}{\sqrt{10^2-4(-5)^2}} = \frac{-10-30}{\sqrt{100-100}} = \frac{-40}{0}$$

→ undef.

$(-)^{\text{even}} = +$, $(-)^{\text{odd}} = -$

Evaluate

1) $(-3)^4 = \boxed{81}$

2) $(-2)^5 = \boxed{-32}$

3) $-4^2 = \boxed{-16}$

4) $-(-3)^2 = -9 = \boxed{-9}$

5) $-(-10^2)$

$= -(-100) = \boxed{100}$

To Simplify expressions, we Combine like terms.

like Terms $\Rightarrow$ Same Variable & Same exponent

$5x^2$, $-7x^2$, $\frac{2}{3}x^2$, $-x^2$, $8y^2$, $12x$

like terms

Unlike terms

$5x^3y^4$, $-12x^4y^3$, $200x^3y$, $45x^2y^4$, ~~$6z^3t^4$~~

Unlike terms

Simplify: $7x + 3x - 4x$

$= 10x - 4x = \boxed{6x}$

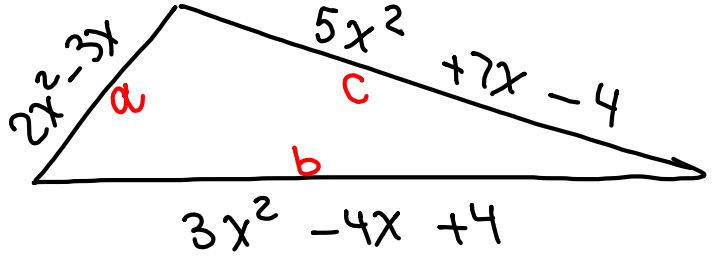
Simplify: $3x^2 + 7y^2 - 5x^2 + 2y^2$

$= \boxed{-2x^2 + 9y^2}$

Simplify: $12x^2 - 7x + 8 - 3x^2 - x - 8$

$= 9x^2 - 8x = \boxed{9x^2 - 8x}$

Find the perimeter: $P = a + b + c$



$P = 2x^2 - 3x + 3x^2 - 4x + 4 + 5x^2 + 7x - 4$

$\boxed{P = 10x^2}$

1) Evaluate $(x-y)^2+2xy$ for $x=2.5$, $y=-3.5$

$=(2.5-(-3.5))^2+2(2.5)(-3.5)$

$=6^2-2(2.5)(3.5)=36-17.5=\boxed{18.5}$

2) Simplify: $25x^3-18x^2+7x-2-5x^3+19x^2-7x+2$

$=20x^3+1x^2=\boxed{20x^3+x^2}$

Properties of Real numbers:

1) Commutative Prop.

$a+b=b+a$

$a\cdot b=b\cdot a$

$7+13=13+7$

$-8\cdot 5=5\cdot(-8)$

2) Associative Prop.

$(a+b)+c=a+(b+c)$

$(a\cdot b)\cdot c=a\cdot(b\cdot c)$

$(x+5)+8=x+(5+8)$

$(4\cdot 3)\cdot 10=4\cdot(3\cdot 10)$

3) Distributive Prop

$a(b+c)=ab+ac$

$a(b-c)=ab-ac$

$4(x+3)=4x+4\cdot 3$

$=4x+12$

$3(2x^2-5x+8)=3(2x^2)-3(5x)+3(8)$

$=6x^2-15x+24$

Distribute & Simplify

$3(2x^2-4x+10)-2(3x^2-6x+15)$

$=6x^2-12x+30-6x^2+12x-30=\boxed{0}$

Identity Prop.

$a+0=a$

$a\cdot 1=a$

$7x+0=7x$

$-4x^2\cdot 1=-4x^2$

Inverse Prop.

$a+(-a)=0$

$a\cdot\frac{1}{a}=1\ ,\ a\neq 0$

$5x^3+(-5x^3)=0$

$25\cdot\frac{1}{25}=1$

Name the properties used:

$4(x+1)-4 = 4x+4\cdot 1-4$ Dist.

$= 4x+4-4$ Identity

$= 4x+0$ Inverse

$= 4x$ Identity

Name the Properties used:

$2\left(\frac{1}{2}x-1\right)+2 = 2\left(\frac{1}{2}x\right)-2\cdot 1+2$ Dist.

$= \left(2\cdot\frac{1}{2}\right)x-2\cdot 1+2$ Assoc.

$= 1x-2+2$ Inverse, Ident.

$= x+0$ Ident. Inverse

$= x$ Identity

Simplify

$$\frac{2}{3}\left(\frac{3}{2}x - 1\right) + \frac{2}{3}$$

$$= \frac{2}{3}\cdot\left(\frac{3}{2}x\right) - \frac{2}{3}\cdot 1 + \frac{2}{3}$$

$$= \left(\frac{2}{3}\cdot\frac{3}{2}\right)x - \frac{2}{3} + \frac{2}{3}$$

$$= 1x - 0$$

$$= \boxed{x}$$

Simplify

$$4(2 + 3x) - 2(4 + 6x)$$

$$= 8 + 12x - 8 - 12x$$

$$= 12x + 8 - 8 - 12x$$

$$= 12x + 0 - 12x$$

$$= 12x - 12x = \boxed{0}$$

Basic Translation:

use Variables for unknowns

The Sum of Some number and 10.

$x + 10$

The difference of 15 and twice Some number.

x

$2x$

$15 - 2x$

3 times Some number increased by Square of the number.

x

$3x + x^2$

4 times the Sum of 10 and Some number reduced by 40.

$4(10 + x) - 40$

-2 times the difference of two numbers increased by the product of the two numbers.

$$-2(x-y)+xy$$

Square root of some number added to 4 times the number squared.

$$4x^2+\sqrt{x}$$

added to
Subtracted from
Reverse them.

Some number cubed subtracted from 3 times the number. Reverse it

$$3x - x^3$$

The quotient of 10 and some number is equal to the number.

$$\frac{10}{x} = x$$

The quotient of some number and 5 more than the number is equal to the ratio of 2 to 3.

$$\frac{x}{x+5} = \frac{2}{3}$$

The number of females in the classroom is 3 fewer than twice the number of males.

-3

Males $\rightarrow x$

Females $\rightarrow 2x - 3$

Draw & label:

Two sides of a triangle are equal.

Third sides is 4 inches shorter than the sum of equal sides.